GCSE Maths – Algebra

Rearranging Formulae

Notes

WORKSHEET

Rearranging Formulae

Mathematical Formulae

A mathematical **formula** is a rule which connects two or more quantities. These quantities are called **variables**. It helps us understand the relationship between them. For example, through a formula we can work out how a change in one **variable** affects the other variables.

Examples of **formula**:

$$Area\ of\ a\ Rectangle = Length \times Width$$
$$Volume\ of\ a\ Cuboid = Length \times\ Width \times Height$$
$$Perimeter\ of\ a\ Rectangle = 2 \times (Length + Width)$$
$$Force = Mass \times Acceleration$$

Variables can also be represented as **letters**.
For example, the above equations can also be written as:

$$A = lw$$
$$V = lwh$$
$$P = 2(l + w)$$
$$F = ma$$

Substitution

Substitution involves swapping letters with numbers to find the value of the expression for desired values.

Example: What is the volume of a cuboid with 5 cm length, 6 cm width and 9 cm height?

If $l = 5, w = 6$ *and* $h = 9$, *then by substitution,*

$$Volume = lwh = l \times w \times h = 5 \times 6 \times 9 = \mathbf{270\ cm^3}$$

Example: You need to convert the temperature from Celsius to Fahrenheit. The temperature is $40°C$ and you have the conversion formula $F = \left(C \times \frac{9}{5}\right) + 32$.

Substituting C into the formula:

$$F = \left(\mathbf{40} \times \frac{9}{5}\right) + 32 = 72 + 32 = 104$$

So the corresponding temperature in degrees Fahrenheit is $\mathbf{104°F}$.

Example: Find the force exerted by an object which has a mass of 56 kg and acceleration of 4 m/s^2 using the relation F = ma.

Substituting $m = 56$ *and* $a = 4$ *into the formula:*

$$F = ma = m \times a = 56 \times 4 = 224$$
$$F = \mathbf{224\ kg\ m/s^2}$$

Rearranging Formulae

Every formula has a **subject**. The subject is the variable which is being computed.

- In the formula $\boldsymbol{F = ma}$, F is the subject, which is the force.
- In the formula for the area of a triangle, $\boldsymbol{A = \frac{1}{2}bh}$, A is the subject which is the area.

However, sometimes we know the subject and instead want to work out one of the other variables. In this case we **rearrange the formula** to change the subject to be the variable we want to work out.

Taking the above examples:

- Making m the subject of $F = ma$ $\rightarrow$ $m = \frac{F}{a}$
- Making h the subject of $A = \frac{1}{2}bh$ $\rightarrow$ $h = \frac{2A}{b}$

Sometimes the rearrangement can be harder and can involve more variables.

Example: Make a the subject of the equation $s = ut + \frac{1}{2}at^2$

1. **Subtract** 'ut' from both sides of the equation.

$$s - ut = \frac{1}{2}at^2$$

2. **Multiply** both sides of the equation by 2.

$$2(s - ut) = at^2$$

3. **Divide** both sides of the equation by t^2.

$$\frac{2}{t^2}(s - ut) = a$$

$$\boldsymbol{a = \frac{2}{t^2}(s - ut)}$$

Example: A trapezium has an area of $60\ \text{m}^2$. The sum of its parallel sides is $24\ \text{m}$.
The formula for the area of a trapezium is

$$A = \frac{a+b}{2} \times h,$$

where $A = area$, $a + b = sum\ of\ parallel\ sides$ and $h = height$.
Find the height of the trapezium.

1. Rearrange the given formula to make h the subject.

   Multiply both sides of the equation by 2 and then divide by $(a + b)$:

$$A = \frac{a+b}{2} \times h$$
$$2A = (a+b) \times h$$
$$h = \frac{2A}{(a+b)}$$

2. Substitute in the given values to find h.

   Substituting $A = 60,\ a + b = 24$:

$$h = \frac{2(60)}{(24)} = \mathbf{5\ m}$$

***Alternative**: Instead of rearranging, we could substitute at the start.*

$$A = \frac{a+b}{2} \times h$$
$$60 = \frac{24}{2} \times h$$
$$60 = 12 \times h$$

Now you can easily work out the height: $h = \frac{60}{12} = \mathbf{5\ m}$

Example: Find the value of q in the expression $a(q - c) = d$, where $a = 2,\ c = 5,\ d = 4$

1. Rearrange the formula to make q the subject of $a(q - c) = d$.

   Divide both sides by a and then add c to both sides:

$$q - c = \frac{d}{a}$$
$$\boldsymbol{q = \frac{d}{a} + c}$$

2. *Substitute the given values.*

   Substituting $a = 2, c = 5, d = 4$: $\quad q = \frac{4}{2} + 5 = \mathbf{7}$

Rearranging harder equations

Fractions in equations make them harder to rearrange. Therefore, when fractions are involved, our first step should be to **eliminate** them, by multiplying both sides of the equation by the denominator of the fraction.

Rearranging equations can also be harder when we have **more than 2 variables**, or when our **subject** is **present in multiple terms**. When this happens, we should try and bring every term with our subject on one side and then factorise.

Example: Make m the subject of the equation $n = \frac{m-1}{2m+7}$.

1. Eliminate the fraction.

   Multiply both sides of the equation by the denominator $(2m + 7)$*:*

   $$n = \frac{m - 1}{2m + 7}$$

   $$n(2m + 7) = m - 1$$

   $$2mn + 7n = m - 1$$

2. Bring all the terms with the required subject to one side of the equation and then factorise the subject out of the terms.

   The required subject is m *so we bring all terms containing* m *to the left-hand side of the equation. Any other terms are moved to the right-hand side of the equation.*

   $$2mn - m = -7n - 1$$

   $$m(2n - 1) = -7n - 1$$

3. Divide by the factored bracket to leave the subject on the left-hand side of the equation.

   $$m(2n - 1) = -7n - 1$$

   $$\boldsymbol{m = \frac{-7n - 1}{2n - 1}}$$

Example: Make m the subject of the equation $\frac{13}{a} = \frac{1}{b-1} - \frac{1}{u}$

1. Eliminate the fraction.

   Multiply the equation by a:

   $$13 = \frac{a}{b-1} - \frac{a}{u}$$

   Multiply the equation by $b-1$:

   $$13(b-1) = a - \frac{a(b-1)}{u}$$

   Multiply the equation by u:

   $$13u(b-1) = au - a(b-1)$$

2. Bring all the terms with the required subject to one side of the equation and then factorise the subject out of the terms.

   $$13u(b-1) - au = -a(b-1)$$

   $$u(13(b-1) - a) = -a(b-1)$$

3. Divide by the factored bracket to leave the subject on the left-hand side of the equation.

   $$u = \frac{-a(b-1)}{13(b-1) - a}$$

   $$\boldsymbol{u = \frac{-ab - a}{13b - 13 - a}}$$

Rearranging Formulae – Practice Questions

1. Using the equation $r = 50d + qm$, find r when $d = 2, q = 6, m = 7$.

2. Using the equation $c = \frac{5}{d} + 4(x + 2)$, find c when $d = 3, x = 10$.

3. Rearrange the following equations to make x the subject:

   a) $y = 3x + 5$

   b) $y = 10(x - 4) + 7$

   c) $y = \frac{6}{x} + 9$

   d) $y = 5(x + z) + 65$

4. Make x the subject of the equation $4y = 7(x + z) + 3y - z$. Then work out x when $y = 42, z = 6$.

5. Rearrange the following formulae to make x the subject.

   a) $7y = \frac{1-4x}{4x+4}$

   b) $y - 3 = \frac{2x+3}{1+3x-3}$

   c) $\frac{1}{r} = \frac{3}{4x} - \frac{5}{7y}$

   d) $\frac{1}{t-6} = \frac{5}{3bx} - \frac{3}{y}$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.